2021

Full Marks - 60

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer *all* questions

Part-I

1. Answer the following : 1×8

a) Find the asymptotes of the curve

$$y^3 = x^2(2a - x).$$

b) Find $\frac{d\bar{v}}{dt}$ at (1, 0, 1) if

$$\bar{v} = \cosh at\ \hat{i} + \sin hat\ \hat{j} + e^{at}\hat{k}.$$

c) Evaluate

$$\int \frac{x\,dx}{x^4 + 16}$$

d) Write nth derivative of log x.

e) Evaluate

$$\lim_{x \to \frac{1}{2}} (\sin x)^{\tan x}$$

f) If $y = \log \sin x$, find the radius of curvature.

g) Evaluate $\int_{2}^{\pi/2} \sin^6 x \, dx$ using Walli's formula.

h) Find the exact arc length of the curve $x = \sin 3t$, $y = \cos 3t$ on the interval $t \in [0, \pi]$.

Part-II

2. Answer any ***eight*** of the following : 1½ × 8

a) Find the whole arc of the region between the curve $xy^2 = a^2 (a - x)$ and y - axis.

b) Evaluate $\int_0^1 \left(e^{-3t}\hat{i} + t^2\hat{j} + \cosh t\,\hat{k}\right) dt$.

c) Evaluate

$$\operatorname*{Lt}_{\theta \to \pi/2} \frac{\log\left(\theta - \pi/2\right)}{\tan \theta}$$

d) Find radius of curvature to the curve $r = a\cos\theta$.

e) Fine the rotation angle θ to remove xy-term in the equation

$$x^2 + 2\sqrt{3}\,xy + 3y^2 - 2x + y - 1 = 0$$

f) Write the reduction formula for

$$\int \sin^x x\,dx.$$

g) Show that the vector valued function $\vec{r}(t) = (2t, 5t, t^2 + t)$ is continuous.

h) Show that the asymptotes of the cubic $x^2y - xy^2 + xy + y^2 + x - y = 0$. Cut the curve again in three points which lie on the line $x + y = 0$.

i) Classify the conic

$$4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$$

j) Find y(t), given

$$y^1(t) = 2t\hat{i} + 3t^2\hat{j},\ y(0) = \hat{i} - 2\hat{j}.$$

Part-III

3. Answer any ***eight*** of the following : 2×8

a) Find the vertical and Horizontal asymptotes of the graph of the function

$$f(x) = \frac{3x+5}{6-x}.$$

b) Find the area of the surface generated by revolving the curve

$y = \sqrt{4-x^2}, -1 \leq x \leq 1$ about x-axis.

c) Using Leibnitz rule to find y_3 where $y = x^3 \sin x$.

d) Evaluate $\underset{x \to \infty}{\mathrm{Lt}} \left(\frac{x}{1+x} \right)^x$.

e) Use reduction formula to evaluate

$$\int \frac{dx}{\left(x^2 + a^2\right)^n}.$$

f) Find the arc length of the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

g) Show that the graph of

$\bar{r}(t)=3(\cos t+1)\hat{i}+2(\sin t+1)\hat{j}$ is an ellipse.

h) Find $\vec{a}.(\vec{b}\times\vec{c})$ if $\vec{a}=2\hat{i}+3\hat{j}+7\hat{k}$, $\vec{b}=\hat{i}+\hat{j}-4\hat{k}$, $\vec{c}=3\hat{i}-2\hat{j}+5\hat{k}$.

i) Trace the graph of $y = 3\sin(2x - 4)$ (without explanation)

j) Find the radius of curvature at (0, 0) for the curve $2x^4 + 3y^4 + 4x^2y + xy - y^2 + 2x = 0$

Part-IV

4. a) Find the nth derivative of $y = e^x \sin^2 x$. 6

OR

b) If $y = \sin(m \sin^{-1}x)$, then prove

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0$$

5. a) Derive the reduction formula for

$\int_0^{\pi/b} e^{ax} \sin bx \, dx$ and hence or otherwise

evaluate $\int_0^{\pi} e^{x} \sin x \, dx$. 6

OR

b) Prove that

$$\int_0^{\pi/2} \log \cos x \, dx = \pi/2 \log\left(\frac{1}{2}\right)$$

6. a) Find the length of the arc of the parabola $x^2 = 4ay$ measured from the vertex to one extremity of the latus rectum. 6

OR

b) Find the volume of the solid generated by revolving around x-axis, the area enclosed by $xy = 4$ and $x + y = 5$, using cyclindrical shell method and washer method.

7. a) If $\vec{F}_1 = t\hat{i} + 4t^2\hat{j} + 5t^3\hat{k}$ and $\vec{F}_2 = 3t^3\hat{i} + 6t^2\hat{j} + 2t\hat{k}$,

then find $\frac{d}{dx}\left(\vec{F}_1 . \vec{F}_2\right)$ and $\frac{d}{dt}\left(\vec{F}_1 \times \vec{F}_2\right)$. 6

OR

b) Find the unit tangent and unit normal to the curve

$$\vec{r}(t) = (3\sin t)\hat{i} + (3\cos t)\hat{j} + 4t\hat{k}.$$

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2021

Full Marks - 80

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Answer *all* questions

Part-I

1. Answer the following : 1×12

a) Find the power set of the set $A=\{\phi, \{\phi\}\}$.

b) Define Zorn's Lemma.

c) Define Chinese remainder theorem.

d) Write the converse, inverse, contrapositive of the statement $p \to q$.

e) How many integers between 1 and 600 divisible by 3 or 5 ?

f) How many functions are there from a set with m elements to a set with n elements ?

g) What is the co-efficient of ${}_{25}x^{12} y^{13}$ in the expansion of $(2x - 3y)^{25}$?

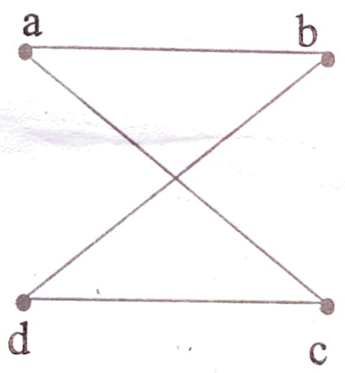
h) Show that the matrix

$$A=\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$$ is a Hermitian matrix.

i) Draw the graph $K_{2,5}$.

j) Let A be the set $\{1,2,3,4\}$. Which ordered pairs are in the relation $R = \{(a, b) | a \text{ divides } b\}$?

k) Write the adJacency matrix of the matrix



l) Show that k_n has a Hamilton circuit wherever $n \geq 3$.

Part-II

2. Answer any ***eight*** of the following : 2 × 8

a) What is the solution of the recurrence relation

$a_n = 6a_{n-1} - 9a_{n-2}$

with initial conditions $a_0 = 1$ and $a_1 = 6$?

b) Let n be the positive integer,

Then prove $\sum_{k=0}^{n}(-1)^k\binom{n}{k}=0$

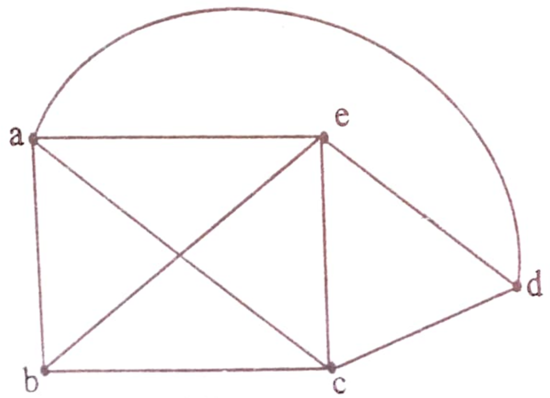
c) Find the rank of the matrix

$$A=\begin{bmatrix}1 & 0 & 1\\ 1 & 2 & 3\\ -1 & 1 & 0\end{bmatrix}$$

d) If λ is an eigen value of the matrix A, Prove that $\frac{1}{\lambda}$ is an eigen value of A^{-1} if A is non-singular.

e) Show that K_5 is not planar.

f) Write the incidence matrix for the graph G.



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g) Show that $n(n+1)(n+5)$ is a multiple of 6.

h) Show that $(p \wedge q) \rightarrow (p \vee q)$ is tautology.

i) Show that the set of positive rational numbers is countable.

j) Use generating functions to find the number of r- combinations from a set with n elements when repetition of elements is allowed.

Part-III

3. Answer any *eight* of the following : 3×8

a) Determine all 2×2 matrices that commute with

$$\begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}.$$

b) How many solutions does $x_1 + x_2 + x_3 = 11$ have, where x_1 , x_2 and x_3 are non-negative integers with $x_1 \leq 3$, $x_2 \leq 4$ and $x_3 \leq 6$?

c) Draw a graph with 4 vertices V_1, V_2, V_3, V_4 such that deg $V_1 = 1$, deg $V_2 = 3$, deg $V_3 = 4$ and deg $V_4 = 2$.

d) Reduce the following matrix to the row-reduced echelon from

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 1 & -1 & 2 \\ 0 & 2 & 1 & -1 \end{bmatrix}$$

e) Suppose that connected planar simple graph has 20 vertices, each of degree 3. How many regions does a representation of this planar graph split the plane ?

f) Let f and g be the functions defined by $f: R \to R^{+} \cup \{0\}$ with $f(x)=x^2$ and $g: R^{+} \cup \{0\} \to R$ with $g(x) = \sqrt{x}$. then find fog(x).

g) Test the consistency and solve the system

$$x + y + z = 6$$
$$x + 2y - 3z = -4$$
$$-x - 4y + 9z = 18$$

h) Find the characteristics polynomial, characterstic equation and the Eigen values of the matrix $\begin{pmatrix} 0 & 3 \\ 2 & -1 \end{pmatrix}$.

i) Prove A graph is non-planar if and only if it contains a subgraph homomorphic to $k_{3,3}, k_5$.

6\.

j) Find the greatest common divisor of 414 and 662 using the Euclidean algorithm.

Part-IV

4\. a) Define Tautology. Construct the Truth Table for $(p \wedge q) \vee (\neg p \wedge r) \vee (q \wedge r)$ 7

OR

b) State Chinese Reminder Theorem.
Solve $x \equiv 1 \pmod 2$, $x \equiv 3 \pmod 5$, $x \equiv 4 \pmod 9$ Using Chinese Theorem.

5\. a) Define principle of mathematical Induction. Using this prove $2^{4x}-1$ is divisible by 15. 7

OR

b) Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with initial conditions $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$.

6. a) Find the range, Kernel, rank and nullity of the matrix 7

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 0 \\ 1 & -1 & -1 \end{pmatrix}$$

OR

b) Solve the followig by row-reduction method.

$$x_1 + 2x_2 + 4x_3 + x_4 = 4$$
$$2x_1 - x_3 + 3x_4 = 4$$
$$x_1 - 2x_2 - x_3 = 0$$
$$3x_1 + x_2 - x_3 - 5x_4 = 7$$

7. a) A graph G has 20 vertices. Any two distinct vertices x and y have the property that deg (x) + deg (y) ≥ 19. Prove that G is connected. 7

OR

b) Let G be connected planar simple graph with a edges and v vertices. Let r be the number of regions in a planar representation of G. then prove $r = e - v + 2$.

2019

Full Marks - 80

Time - 3 hours

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Answer *all* questions

1. a) Define asymptote. Find all the asymptotes of the curve $x^3y - y^4 - 3x^2y^2 + 3xy^3 - xy + y^2 - 2 = 0$. 10

   b) Find the area of the circle $x^2 + y^2 = 16$ using quadrature. 3

   c) Find the curvature of the curve 3
   $x^3 + y^3 + x^2 + y^2 + y = 0$

   OR

   d) Find the radius of curvature of the curve $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$ where it touches x-axis. 10

   e) Find the asymptote parallel to y-axis of the curve $x^2y^2 - a^2(x^2 + y^2) = 0$. 3

   f) Find the volume of a sphere of radius a by revolving the circle $x^2 + y^2 = a^2$ about x-axis. 3

2. a) Prove that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact. 10

b) Identify the surface $\frac{x^2}{4} + y^2 + \frac{z^2}{4} = 1$ and find its intercept on y-axis. 3

c) Find the points of intersection of the conicoid $12x^2 - 17y^2 + 7z^2 = 7$and the line

$$\frac{x+5}{-3} = \frac{y-4}{1} = \frac{z-11}{7}.$$ 3

OR

d) Find the equation of the enveloping cylinder on the shpere $x^2 + y^2 + z^2 = 4$ and whose generations are parallel to the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$. 10

e) Find the radius of the circle given by the equation $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0 = x + 2y + 2z - 15$. 3

f) Prove that the plane $24x + 51y - 28z + 7 = 0$ touches the conicoid $12x^2 - 17y^2 + 7z^2 = 7$. 3

3. a) Examine the continuity of f(x, y) at (0, 0) where

$$f(x, y) = \begin{cases} \frac{xy^2}{x^4 + y^4}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$ 10

b) If $f(x, y) = \frac{x+y}{x-y}$, them find $f_x(2, 1)$. 3

c) If $z = \ln r$ wher $r^2 = (x-a)^2 + (y-b)^2$, then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$. 3

OR

d) Evaluate $\iint_R (x + y)\, dy\, dx$, where R in the region bounded by $x = 0, x = 2, y = x, y = x + 2$. 10

e) Expand $f(x, y) = x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ by Taylor's theorem. 3

f) Find $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2 + y^2}$. 3

4. a) What is homogenous differential equation ? Solve $(x^2 - 2y^2)\, dx + xy\, dy = 0$. 10

b) Solve : $p^2 - 2xp + 1 = 0$ where $p = \frac{dy}{dx}$. 3

c) Find an integrating factor of 3
$(3x^2 - y^2)\, dy - 2xy\, dx = 0$

OR

d) What is exact differential equation ? Solve the IVP :
$(\cos x + y \sin x)dx - \cos x\, dy = 0,\ y(\pi) = 0.$ 10

e) Solve : $xy\, dx + (x + 1)\, dy = 0.$ 3

f) Solve : $x = p^3 - p + 2$, where $p = \frac{dy}{dx}$. 3

5. a) Solve : $(D^2 - 4D + 3)y = e^x \cos 2x + \cos 3x.$ 10

b) Solve : $(D^4 + 2D^2 + 1)\, y = 0.$ 3

c) Find the Laplace transform of
$f(t) = 3 \sin ut - 2 \cos 5t.$ 3

OR

d) Solve : $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x.$ 10

e) Find particular integral of $(D^2 + 1) = \operatorname{cosec} x$ by method of variation of parameter. 3

f) Find $L^{-1}\left[\frac{p+1}{p^3 + p^2 - 6p}\right]$. 3